

18

Superposition and Standing Waves

CHAPTER OUTLINE

- 18.1 Superposition and Interference
- 18.2 Standing Waves
- 18.3 Standing Waves in a String Fixed at Both Ends
- 18.4 Resonance
- 18.5 Standing Waves in Air Columns
- 18.6 Standing Waves in Rod and Plates
- 18.7 Beats: Interference in Time
- 18.8 Non-Sinusoidal Wave Patterns

ANSWERS TO QUESTIONS

- Q18.1** No. Waves with other waveforms are also trains of disturbance that add together when waves from different sources move through the same medium at the same time.
- Q18.2** The energy has not disappeared, but is still carried by the wave pulses. Each particle of the string still has kinetic energy. This is similar to the motion of a simple pendulum. The pendulum does not stop at its equilibrium position during oscillation—likewise the particles of the string do not stop at the equilibrium position of the string when these two waves superimpose.
- Q18.3** No. A wave is not a solid object, but a chain of disturbance. As described by the principle of superposition, the waves move through each other.
- Q18.4** They can, wherever the two waves are nearly enough in phase that their displacements will add to create a total displacement greater than the amplitude of either of the two original waves.
When two one-dimensional sinusoidal waves of the same amplitude interfere, this condition is satisfied whenever the absolute value of the phase difference between the two waves is less than 120° .
- Q18.5** When the two tubes together are not an efficient transmitter of sound from source to receiver, they are an efficient reflector. The incoming sound is reflected back to the source. The waves reflected by the two tubes separately at the junction interfere constructively.
- Q18.6** No. The total energy of the pair of waves remains the same. Energy missing from zones of destructive interference appears in zones of constructive interference.
- Q18.7** Each of these standing wave patterns is made of two superimposed waves of identical frequencies traveling, and hence transferring energy, in opposite directions. Since the energy transfer of the waves are equal, then no net transfer of energy occurs.
- Q18.8** Damping, and non-linear effects in the vibration turn the energy of vibration into internal energy.
- Q18.9** The air in the shower stall can vibrate in standing wave patterns to intensify those frequencies in your voice which correspond to its free vibrations. The hard walls of the bathroom reflect sound very well to make your voice more intense at all frequencies, giving the room a longer reverberation time. The reverberant sound may help you to stay on key.

- Q18.10** The trombone slide and trumpet valves change the length of the air column inside the instrument, to change its resonant frequencies.
- Q18.11** The vibration of the air must have zero amplitude at the closed end. For air in a pipe closed at one end, the diagrams show how resonance vibrations have NA distances that are odd integer submultiples of the NA distance in the fundamental vibration. If the pipe is open, resonance vibrations have NA distances that are all integer submultiples of the NA distance in the fundamental.

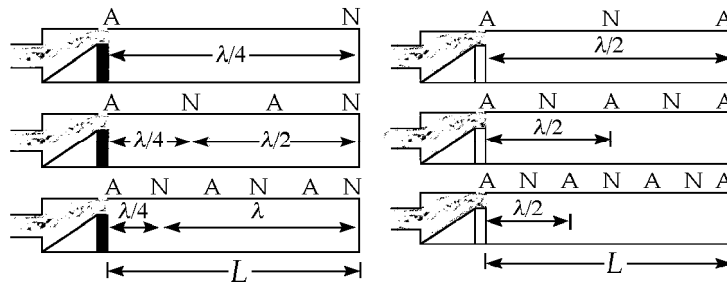


FIG. Q18.11

- Q18.12** What is needed is a tuning fork—or other pure-tone generator—of the desired frequency. Strike the tuning fork and pluck the corresponding string on the piano at the same time. If they are precisely in tune, you will hear a single pitch with no amplitude modulation. If the two pitches are a bit off, you will hear beats. As they vibrate, retune the piano string until the beat frequency goes to zero.
- Q18.13** Air blowing fast by a rim of the pipe creates a “shshshsh” sound called edgetone noise, a mixture of all frequencies, as the air turbulently switches between flowing on one side of the edge and the other. The air column inside the pipe finds one or more of its resonance frequencies in the noise. The air column starts vibrating with large amplitude in a standing wave vibration mode. It radiates sound into the surrounding air (and also locks the flapping airstream at the edge to its own frequency, making the noise disappear after just a few cycles).
- Q18.14** A typical standing-wave vibration possibility for a bell is similar to that for the glass shown in Figure 18.17. Here six node-to-node distances fit around the circumference of the rim. The circumference is equal to three times the wavelength of the transverse wave of in-and-out bending of the material. In other states the circumference is two, four, five, or higher integers times the wavelengths of the higher-frequency vibrations. (The circumference being equal to the wavelength would describe the bell moving from side to side without bending, which it can do without producing any sound.) A tuned bell is cast and shaped so that some of these vibrations will have their frequencies constitute higher harmonics of a musical note, the strike tone. This tuning is lost if a crack develops in the bell. The sides of the crack vibrate as antinodes. Energy of vibration may be rapidly converted into internal energy at the end of the crack, so the bell may not ring for so long a time.
- Q18.15** The bow string is pulled away from equilibrium and released, similar to the way that a guitar string is pulled and released when it is plucked. Thus, standing waves will be excited in the bow string. If the arrow leaves from the exact center of the string, then a series of odd harmonics will be excited. Even harmonics will not be excited because they have a node at the point where the string exhibits its maximum displacement.

- Q18.16** Walking makes the person's hand vibrate a little. If the frequency of this motion is equal to the natural frequency of coffee sloshing from side to side in the cup, then a large-amplitude vibration of the coffee will build up in resonance. To get off resonance and back to the normal case of a small-amplitude disturbance producing a small-amplitude result, the person can walk faster, walk slower, or get a larger or smaller cup. Alternatively, even at resonance he can reduce the amplitude by adding damping, as by stirring high-fiber quick-cooking oatmeal into the hot coffee.
- Q18.17** Beats. The propellers are rotating at slightly different frequencies.
- Q18.18** Instead of just radiating sound very softly into the surrounding air, the tuning fork makes the chalkboard vibrate. With its large area this stiff sounding board radiates sound into the air with higher power. So it drains away the fork's energy of vibration faster and the fork stops vibrating sooner. This process exemplifies conservation of energy, as the energy of vibration of the fork is transferred through the blackboard into energy of vibration of the air.
- Q18.19** The difference between static and kinetic friction makes your finger alternately slip and stick as it slides over the glass. Your finger produces a noisy vibration, a mixture of different frequencies, like new sneakers on a gymnasium floor. The glass finds one of its resonance frequencies in the noise. The thin stiff wall of the cup starts vibrating with large amplitude in a standing wave vibration mode. A typical possibility is shown in Figure 18.17. It radiates sound into the surrounding air, and also can lock your squeaking finger to its own frequency, making the noise disappear after just a few cycles. Get a lot of different thin-walled glasses of fine crystal and try them out. Each will generally produce a different note. You can tune them by adding wine.
- Q18.20** Helium is less dense than air. It carries sound at higher speed. Each cavity in your vocal apparatus has a standing-wave resonance frequency, and each of these frequencies is shifted to a higher value. Your vocal chords can vibrate at the same fundamental frequency, but your vocal tract amplifies by resonance a different set of higher frequencies. Then your voice has a different quacky quality.
Warning: Inhaling any pressurized gas can cause a gas embolism which can lead to stroke or death, regardless of your age or health status. If you plan to try this demonstration in class, inhale your helium from a balloon, not directly from a pressurized tank.
- Q18.21** Stick a bit of chewing gum to one tine of the second fork. If the beat frequency is then faster than 4 beats per second, the second has a lower frequency than the standard fork. If the beats have slowed down, the second fork has a higher frequency than the standard. Remove the gum, clean the fork, add or subtract 4 Hz according to what you found, and your answer will be the frequency of the second fork.

SOLUTIONS TO PROBLEMS

Section 18.1 Superposition and Interference

P18.1 $y = y_1 + y_2 = 3.00 \cos(4.00x - 1.60t) + 4.00 \sin(5.0x - 2.00t)$ evaluated at the given x values.

(a) $x = 1.00, t = 1.00$ $y = 3.00 \cos(2.40 \text{ rad}) + 4.00 \sin(+3.00 \text{ rad}) = \boxed{-1.65 \text{ cm}}$

(b) $x = 1.00, t = 0.500$ $y = 3.00 \cos(+3.20 \text{ rad}) + 4.00 \sin(+4.00 \text{ rad}) = \boxed{-6.02 \text{ cm}}$

(c) $x = 0.500, t = 0$ $y = 3.00 \cos(+2.00 \text{ rad}) + 4.00 \sin(+2.50 \text{ rad}) = \boxed{1.15 \text{ cm}}$

P18.2

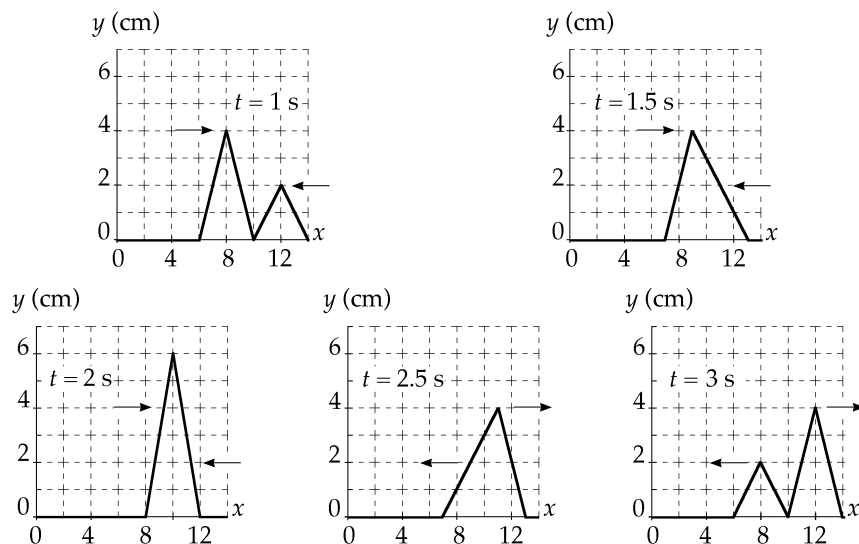


FIG. P18.2

P18.3 (a) $y_1 = f(x - vt)$, so wave 1 travels in the +x direction

$y_2 = f(x + vt)$, so wave 2 travels in the -x direction

(b) To cancel, $y_1 + y_2 = 0$:

$$\frac{5}{(3x - 4t)^2 + 2} = \frac{+5}{(3x + 4t - 6)^2 + 2}$$

$$(3x - 4t)^2 = (3x + 4t - 6)^2$$

$$3x - 4t = \pm(3x + 4t - 6)$$

for the positive root, $8t = 6$

$t = 0.750 \text{ s}$

(at $t = 0.750 \text{ s}$, the waves cancel everywhere)

(c) for the negative root, $6x = 6$

$x = 1.00 \text{ m}$

(at $x = 1.00 \text{ m}$, the waves cancel always)

P18.4 Suppose the waves are sinusoidal.

The sum is $(4.00 \text{ cm})\sin(kx - \omega t) + (4.00 \text{ cm})\sin(kx - \omega t + 90.0^\circ)$

$$2(4.00 \text{ cm})\sin(kx - \omega t + 45.0^\circ)\cos 45.0^\circ$$

So the amplitude is $(8.00 \text{ cm})\cos 45.0^\circ =$ 5.66 cm .

P18.5 The resultant wave function has the form

$$y = 2A_0 \cos\left(\frac{\phi}{2}\right)\sin\left(kx - \omega t + \frac{\phi}{2}\right)$$

(a) $A = 2A_0 \cos\left(\frac{\phi}{2}\right) = 2(5.00)\cos\left[\frac{-\pi/4}{2}\right] =$ 9.24 m

(b) $f = \frac{\omega}{2\pi} = \frac{1200\pi}{2\pi} =$ 600 Hz

P18.6 $2A_0 \cos\left(\frac{\phi}{2}\right) = A_0$ so

$$\frac{\phi}{2} = \cos^{-1}\left(\frac{1}{2}\right) = 60.0^\circ = \frac{\pi}{3}$$

Thus, the phase difference is

$$\phi = 120^\circ = \frac{2\pi}{3}$$

This phase difference results if the time delay is

$$\frac{T}{3} = \frac{1}{3f} = \frac{\lambda}{3v}$$

$$\text{Time delay} = \frac{3.00 \text{ m}}{3(2.00 \text{ m/s})} = \boxed{0.500 \text{ s}}$$

P18.7 (a) If the end is fixed, there is inversion of the pulse upon reflection. Thus, when they meet, they cancel and the amplitude is $\boxed{\text{zero}}$.

(b) If the end is free, there is no inversion on reflection. When they meet, the amplitude is $2A = 2(0.150 \text{ m}) = \boxed{0.300 \text{ m}}$.

P18.8 (a) $\Delta x = \sqrt{9.00 + 4.00} - 3.00 = \sqrt{13} - 3.00 = 0.606 \text{ m}$

The wavelength is $\lambda = \frac{v}{f} = \frac{343 \text{ m/s}}{300 \text{ Hz}} = 1.14 \text{ m}$

Thus, $\frac{\Delta x}{\lambda} = \frac{0.606}{1.14} = 0.530$ of a wave,

or $\Delta\phi = 2\pi(0.530) = \boxed{3.33 \text{ rad}}$

(b) For destructive interference, we want $\frac{\Delta x}{\lambda} = 0.500 = f \frac{\Delta x}{v}$

where Δx is a constant in this set up. $f = \frac{v}{2\Delta x} = \frac{343}{2(0.606)} = \boxed{283 \text{ Hz}}$

P18.9 We suppose the man's ears are at the same level as the lower speaker. Sound from the upper speaker is delayed by traveling the extra distance $\sqrt{L^2 + d^2} - L$.

He hears a minimum when this is $\frac{(2n-1)\lambda}{2}$ with $n = 1, 2, 3, \dots$

Then, $\sqrt{L^2 + d^2} - L = \frac{(n-1/2)v}{f}$

$$\sqrt{L^2 + d^2} = \frac{(n-1/2)v}{f} + L$$

$$L^2 + d^2 = \frac{(n-1/2)^2 v^2}{f^2} + L^2 + \frac{2(n-1/2)vL}{f}$$

$$L = \frac{d^2 - (n-1/2)^2 v^2 / f^2}{2(n-1/2)v/f} \quad n = 1, 2, 3, \dots$$

This will give us the answer to (b). The path difference starts from nearly zero when the man is very far away and increases to d when $L = 0$. The number of minima he hears is the greatest integer

solution to $d \geq \frac{(n-1/2)v}{f}$

$$n = \text{greatest integer} \leq \frac{df}{v} + \frac{1}{2}.$$

continued on next page

$$(a) \quad \frac{df}{v} + \frac{1}{2} = \frac{(4.00 \text{ m})(200/\text{s})}{330 \text{ m/s}} + \frac{1}{2} = 2.92$$

He hears two minima.

(b) With $n = 1$,

$$L = \frac{d^2 - (1/2)^2 v^2 / f^2}{2(1/2)v/f} = \frac{(4.00 \text{ m})^2 - (330 \text{ m/s})^2 / 4(200/\text{s})^2}{(330 \text{ m/s})/200/\text{s}}$$

$$L = \boxed{9.28 \text{ m}}$$

with $n = 2$

$$L = \frac{d^2 - (3/2)^2 v^2 / f^2}{2(3/2)v/f} = \boxed{1.99 \text{ m}}.$$

P18.10 Suppose the man's ears are at the same level as the lower speaker. Sound from the upper speaker is delayed by traveling the extra distance $\Delta r = \sqrt{L^2 + d^2} - L$.

He hears a minimum when $\Delta r = (2n - 1)\left(\frac{\lambda}{2}\right)$ with $n = 1, 2, 3, \dots$

$$\begin{aligned} \text{Then, } \quad \sqrt{L^2 + d^2} - L &= \left(n - \frac{1}{2}\right)\left(\frac{v}{f}\right) \\ \sqrt{L^2 + d^2} &= \left(n - \frac{1}{2}\right)\left(\frac{v}{f}\right) + L \\ L^2 + d^2 &= \left(n - \frac{1}{2}\right)^2 \left(\frac{v}{f}\right)^2 + 2\left(n - \frac{1}{2}\right)\left(\frac{v}{f}\right)L + L^2 \\ d^2 - \left(n - \frac{1}{2}\right)^2 \left(\frac{v}{f}\right)^2 &= 2\left(n - \frac{1}{2}\right)\left(\frac{v}{f}\right)L \end{aligned} \tag{1}$$

Equation 1 gives the distances from the lower speaker at which the man will hear a minimum. The path difference Δr starts from nearly zero when the man is very far away and increases to d when $L = 0$.

(a) The number of minima he hears is the greatest integer value for which $L \geq 0$. This is the same as the greatest integer solution to $d \geq \left(n - \frac{1}{2}\right)\left(\frac{v}{f}\right)$, or

$$\boxed{\text{number of minima heard} = n_{\max} = \text{greatest integer} \leq d\left(\frac{f}{v}\right) + \frac{1}{2}}.$$

(b) From equation 1, the distances at which minima occur are given by

$$\boxed{L_n = \frac{d^2 - (n - 1/2)^2 (v/f)^2}{2(n - 1/2)(v/f)} \text{ where } n = 1, 2, \dots, n_{\max}}.$$

P18.11 (a) $\phi_1 = (20.0 \text{ rad/cm})(5.00 \text{ cm}) - (32.0 \text{ rad/s})(2.00 \text{ s}) = 36.0 \text{ rad}$
 $\phi_2 = (25.0 \text{ rad/cm})(5.00 \text{ cm}) - (40.0 \text{ rad/s})(2.00 \text{ s}) = 45.0 \text{ rad}$
 $\Delta\phi = 9.00 \text{ radians} = 516^\circ = \boxed{156^\circ}$

(b) $\Delta\phi = |20.0x - 32.0t - [25.0x - 40.0t]| = |-5.00x + 8.00t|$

At $t = 2.00 \text{ s}$, the requirement is

$$\Delta\phi = |-5.00x + 8.00(2.00)| = (2n+1)\pi \text{ for any integer } n.$$

For $x < 3.20$, $-5.00x + 16.0$ is positive, so we have

$$-5.00x + 16.0 = (2n+1)\pi, \text{ or}$$

$$x = 3.20 - \frac{(2n+1)\pi}{5.00}$$

The smallest positive value of x occurs for $n = 2$ and is

$$x = 3.20 - \frac{(4+1)\pi}{5.00} = 3.20 - \pi = \boxed{0.0584 \text{ cm}}.$$

P18.12 (a) First we calculate the wavelength: $\lambda = \frac{v}{f} = \frac{344 \text{ m/s}}{21.5 \text{ Hz}} = 16.0 \text{ m}$

Then we note that the path difference equals $9.00 \text{ m} - 1.00 \text{ m} = \boxed{\frac{1}{2}\lambda}$

Therefore, the receiver will record a minimum in sound intensity.

(b) We choose the origin at the midpoint between the speakers. If the receiver is located at point (x, y) , then we must solve:

$$\sqrt{(x+5.00)^2 + y^2} - \sqrt{(x-5.00)^2 + y^2} = \frac{1}{2}\lambda$$

Then,

$$\sqrt{(x+5.00)^2 + y^2} = \sqrt{(x-5.00)^2 + y^2} + \frac{1}{2}\lambda$$

Square both sides and simplify to get:

$$20.0x - \frac{\lambda^2}{4} = \lambda\sqrt{(x-5.00)^2 + y^2}$$

Upon squaring again, this reduces to:

$$400x^2 - 10.0\lambda^2x + \frac{\lambda^4}{16.0} = \lambda^2(x-5.00)^2 + \lambda^2y^2$$

Substituting $\lambda = 16.0 \text{ m}$, and reducing,

$$\boxed{9.00x^2 - 16.0y^2 = 144}$$

or

$$\frac{x^2}{16.0} - \frac{y^2}{9.00} = 1$$

(When plotted this yields a curve called a hyperbola.)

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Section 18.2 **Standing Waves**

P18.13 $y = (1.50 \text{ m}) \sin(0.400x) \cos(200t) = 2A_0 \sin kx \cos \omega t$

Therefore, $k = \frac{2\pi}{\lambda} = 0.400 \text{ rad/m}$ $\lambda = \frac{2\pi}{0.400 \text{ rad/m}} = \boxed{15.7 \text{ m}}$

and $\omega = 2\pi f$ so $f = \frac{\omega}{2\pi} = \frac{200 \text{ rad/s}}{2\pi \text{ rad}} = \boxed{31.8 \text{ Hz}}$

The speed of waves in the medium is $v = \lambda f = \frac{\lambda}{2\pi} 2\pi f = \frac{\omega}{k} = \frac{200 \text{ rad/s}}{0.400 \text{ rad/m}} = \boxed{500 \text{ m/s}}$

P18.14 $y = 0.0300 \text{ m} \cos\left(\frac{x}{2}\right) \cos(40t)$

(a) nodes occur where $y = 0$:

$$\frac{x}{2} = (2n+1)\frac{\pi}{2}$$

so $x = \boxed{(2n+1)\pi = \pi, 3\pi, 5\pi, \dots}$.

(b) $y_{\max} = 0.0300 \text{ m} \cos\left(\frac{0.400}{2}\right) = \boxed{0.0294 \text{ m}}$

P18.15 The facing speakers produce a standing wave in the space between them, with the spacing between nodes being

$$d_{\text{NN}} = \frac{\lambda}{2} = \frac{v}{2f} = \frac{343 \text{ m/s}}{2(800 \text{ s}^{-1})} = 0.214 \text{ m}$$

If the speakers vibrate in phase, the point halfway between them is an antinode of pressure at a distance from either speaker of

$$\frac{1.25 \text{ m}}{2} = 0.625.$$

Then there is a node at $0.625 - \frac{0.214}{2} = \boxed{0.518 \text{ m}}$

a node at $0.518 \text{ m} - 0.214 \text{ m} = \boxed{0.303 \text{ m}}$

a node at $0.303 \text{ m} - 0.214 \text{ m} = \boxed{0.0891 \text{ m}}$

a node at $0.518 \text{ m} + 0.214 \text{ m} = \boxed{0.732 \text{ m}}$

a node at $0.732 \text{ m} + 0.214 \text{ m} = \boxed{0.947 \text{ m}}$

and a node at $0.947 \text{ m} + 0.214 \text{ m} = \boxed{1.16 \text{ m}}$ from either speaker.

P18.16 $y = 2A_0 \sin kx \cos \omega t$

$$\frac{\partial^2 y}{\partial x^2} = -2A_0 k^2 \sin kx \cos \omega t$$

$$\frac{\partial^2 y}{\partial t^2} = -2A_0 \omega^2 \sin kx \cos \omega t$$

Substitution into the wave equation gives $-2A_0 k^2 \sin kx \cos \omega t = \left(\frac{1}{v^2}\right)(-2A_0 \omega^2 \sin kx \cos \omega t)$

This is satisfied, provided that $v = \frac{\omega}{k}$

P18.17 $y_1 = 3.00 \sin[\pi(x + 0.600t)] \text{ cm}; y_2 = 3.00 \sin[\pi(x - 0.600t)] \text{ cm}$

$$y = y_1 + y_2 = [3.00 \sin(\pi x) \cos(0.600\pi t) + 3.00 \sin(\pi x) \cos(0.600\pi t)] \text{ cm}$$

$$y = (6.00 \text{ cm}) \sin(\pi x) \cos(0.600\pi t)$$

(a) We can take $\cos(0.600\pi t) = 1$ to get the maximum y .

At $x = 0.250 \text{ cm}$, $y_{\max} = (6.00 \text{ cm}) \sin(0.250\pi) = \boxed{4.24 \text{ cm}}$

(b) At $x = 0.500 \text{ cm}$, $y_{\max} = (6.00 \text{ cm}) \sin(0.500\pi) = \boxed{6.00 \text{ cm}}$

(c) Now take $\cos(0.600\pi t) = -1$ to get $y_{\max}$:

At $x = 1.50 \text{ cm}$, $y_{\max} = (6.00 \text{ cm}) \sin(1.50\pi)(-1) = \boxed{6.00 \text{ cm}}$

(d) The antinodes occur when $x = \frac{n\lambda}{4} \ (n = 1, 3, 5, \dots)$

But $k = \frac{2\pi}{\lambda} = \pi$, so $\lambda = 2.00 \text{ cm}$

and $x_1 = \frac{\lambda}{4} = \boxed{0.500 \text{ cm}}$ as in (b)

$$x_2 = \frac{3\lambda}{4} = \boxed{1.50 \text{ cm}}$$
 as in (c)

$$x_3 = \frac{5\lambda}{4} = \boxed{2.50 \text{ cm}}$$

P18.18 (a) The resultant wave is $y = 2A \sin\left(kx + \frac{\phi}{2}\right) \cos\left(\omega t - \frac{\phi}{2}\right)$

The nodes are located at $kx + \frac{\phi}{2} = n\pi$

so $x = \frac{n\pi}{k} - \frac{\phi}{2k}$

which means that each node is shifted $\frac{\phi}{2k}$ to the left.

(b) The separation of nodes is $\Delta x = \left[(n+1)\frac{\pi}{k} - \frac{\phi}{2k}\right] - \left[\frac{n\pi}{k} - \frac{\phi}{2k}\right]$

$$\Delta x = \frac{\pi}{k} = \frac{\lambda}{2}$$

The nodes are still separated by half a wavelength.

Section 18.3 Standing Waves in a String Fixed at Both Ends

P18.19 $L = 30.0 \text{ m}$; $\mu = 9.00 \times 10^{-3} \text{ kg/m}$; $T = 20.0 \text{ N}$; $f_1 = \frac{v}{2L}$

where $v = \left(\frac{T}{\mu}\right)^{1/2} = 47.1 \text{ m/s}$

so $f_1 = \frac{47.1}{60.0} = \boxed{0.786 \text{ Hz}}$ $f_2 = 2f_1 = \boxed{1.57 \text{ Hz}}$

$f_3 = 3f_1 = \boxed{2.36 \text{ Hz}}$ $f_4 = 4f_1 = \boxed{3.14 \text{ Hz}}$

***P18.20** The tension in the string is

$$T = (4 \text{ kg})(9.8 \text{ m/s}^2) = 39.2 \text{ N}$$

Its linear density is

$$\mu = \frac{m}{L} = \frac{8 \times 10^{-3} \text{ kg}}{5 \text{ m}} = 1.6 \times 10^{-3} \text{ kg/m}$$

and the wave speed on the string is

$$v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{39.2 \text{ N}}{1.6 \times 10^{-3} \text{ kg/m}}} = 156.5 \text{ m/s}$$

In its fundamental mode of vibration, we have

$$\lambda = 2L = 2(5 \text{ m}) = 10 \text{ m}$$

Thus,

$$f = \frac{v}{\lambda} = \frac{156.5 \text{ m/s}}{10 \text{ m}} = \boxed{15.7 \text{ Hz}}$$

P18.21 (a) Let n be the number of nodes in the standing wave resulting from the 25.0-kg mass. Then $n + 1$ is the number of nodes for the standing wave resulting from the 16.0-kg mass. For standing waves, $\lambda = \frac{2L}{n}$, and the frequency is $f = \frac{v}{\lambda}$.

Thus, $f = \frac{n}{2L} \sqrt{\frac{T_n}{\mu}}$

and also $f = \frac{n+1}{2L} \sqrt{\frac{T_{n+1}}{\mu}}$

Thus, $\frac{n+1}{n} = \sqrt{\frac{T_n}{T_{n+1}}} = \sqrt{\frac{(25.0 \text{ kg})g}{(16.0 \text{ kg})g}} = \frac{5}{4}$

Therefore, $4n + 4 = 5n$, or $n = 4$

Then, $f = \frac{4}{2(2.00 \text{ m})} \sqrt{\frac{(25.0 \text{ kg})(9.80 \text{ m/s}^2)}{0.00200 \text{ kg/m}}} = \boxed{350 \text{ Hz}}$

(b) The largest mass will correspond to a standing wave of 1 loop

($n = 1$) so $350 \text{ Hz} = \frac{1}{2(2.00 \text{ m})} \sqrt{\frac{m(9.80 \text{ m/s}^2)}{0.00200 \text{ kg/m}}}$

yielding $m = \boxed{400 \text{ kg}}$

***P18.22** The first string has linear density

$$\mu_1 = \frac{1.56 \times 10^{-3} \text{ kg}}{0.658 \text{ m}} = 2.37 \times 10^{-3} \text{ kg/m}.$$

$$\text{The second, } \mu_2 = \frac{6.75 \times 10^{-3} \text{ kg}}{0.950 \text{ m}} = 7.11 \times 10^{-3} \text{ kg/m}.$$

The tension in both is $T = 6.93 \text{ kg } 9.8 \text{ m/s}^2 = 67.9 \text{ N}$. The speed of waves in the first string is

$$v_1 = \sqrt{\frac{T}{\mu_1}} = \sqrt{\frac{67.9 \text{ N}}{2.37 \times 10^{-3} \text{ kg/m}}} = 169 \text{ m/s}$$

and in the second $v_2 = \sqrt{\frac{T}{\mu_2}} = 97.8 \text{ m/s}$. The two strings vibrate at the same frequency, according to

$$\frac{n_1 v_1}{2L_1} = \frac{n_2 v_2}{2L_2}$$

$$\frac{n_1 169 \text{ m/s}}{2(0.658 \text{ m})} = \frac{n_2 97.8 \text{ m/s}}{2(0.950 \text{ m})}$$

$\frac{n_2}{n_1} = 2.50 = \frac{5}{2}$. Thus $n_1 = 2$ and $n_2 = 5$ are the number of antinodes on each string in the lowest resonance with a node at the junction.

- (b) The first string has $2 + 1 = 3$ nodes and the second string 5 more nodes, for a total of 8, or 6 other than the vibrator and pulley.

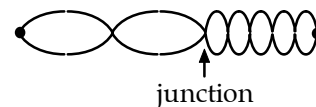


FIG. P18.22(b)

- (a) The frequency is $\frac{2(169 \text{ m/s})}{2(0.658 \text{ m})} = \text{span style="border: 1px solid black; padding: 0 5px;">257 \text{ Hz}.$

***P18.23** For the E-string on a guitar vibrating as a whole, $v = f\lambda = 330 \text{ Hz}(2)64.0 \text{ cm}$. When it is stopped at the first fret we have $\sqrt[12]{2} 330 \text{ Hz}(2)L_F = v = 330 \text{ Hz}(2)64.0 \text{ cm}$. So $L_F = \frac{64.0 \text{ cm}}{\sqrt[12]{2}}$. Similarly for the second fret, $2^{2/12} 330 \text{ Hz}(2)L_{F\#} = v = 330 \text{ Hz}(2)64.0 \text{ cm}$. $L_{F\#} = \frac{64.0 \text{ cm}}{2^{2/12}}$. The spacing between the first and second frets is

$$64.0 \text{ cm} \left(\frac{1}{2^{1/12}} - \frac{1}{2^{2/12}} \right) = 64.0 \text{ cm} \left(\frac{1}{1.0595} - \frac{1}{1.0595^2} \right) = 3.39 \text{ cm}.$$

This is a more precise version of the answer to the example in the text.

Now the eighteenth fret is distant from the bridge by $L_{18} = \frac{64.0 \text{ cm}}{2^{18/12}}$. And the nineteenth lets this much string vibrate: $L_{19} = \frac{64.0 \text{ cm}}{2^{19/12}}$. The distance between them is

$$64.0 \text{ cm} \left(\frac{1}{2^{18/12}} - \frac{1}{2^{19/12}} \right) = 64.0 \text{ cm} \frac{1}{2^{1.5}} \left(1 - \frac{1}{2^{1/12}} \right) = \text{span style="border: 1px solid black; padding: 0 5px;">1.27 \text{ cm}.$$

***P18.24** For the whole string vibrating, $d_{NN} = 0.64 \text{ m} = \frac{\lambda}{2}$; $\lambda = 1.28 \text{ m}$. The speed of a pulse on the string is $v = f\lambda = 330 \frac{1}{s} 1.28 \text{ m} = 422 \text{ m/s}$.

- (a) When the string is stopped at the fret, $d_{NN} = \frac{2}{3} 0.64 \text{ m} = \frac{\lambda}{2}$;
 $\lambda = 0.853 \text{ m}$

$$f = \frac{v}{\lambda} = \frac{422 \text{ m/s}}{0.853 \text{ m}} = \boxed{495 \text{ Hz}}.$$

- (b) The light touch at a point one third of the way along the string damps out vibration in the two lowest vibration states of the string as a whole. The whole string vibrates in its third resonance possibility: $3d_{NN} = 0.64 \text{ m} = 3 \frac{\lambda}{2}$;
 $\lambda = 0.427 \text{ m}$

$$f = \frac{v}{\lambda} = \frac{422 \text{ m/s}}{0.427 \text{ m}} = \boxed{990 \text{ Hz}}.$$

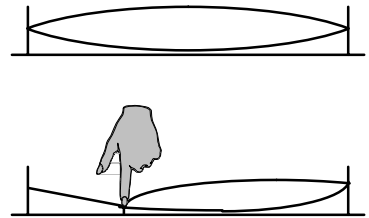


FIG. P18.24(a)

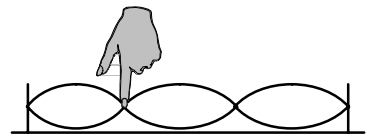


FIG. P18.24(b)

P18.25 $f_1 = \frac{v}{2L}$, where $v = \left(\frac{T}{\mu}\right)^{1/2}$

- (a) If L is doubled, then $f_1 \propto L^{-1}$ will be reduced by a factor $\frac{1}{2}$.
 (b) If μ is doubled, then $f_1 \propto \mu^{-1/2}$ will be reduced by a factor $\frac{1}{\sqrt{2}}$.
 (c) If T is doubled, then $f_1 \propto \sqrt{T}$ will increase by a factor of $\sqrt{2}$.

P18.26 $L = 60.0 \text{ cm} = 0.600 \text{ m}$; $T = 50.0 \text{ N}$; $\mu = 0.100 \text{ g/cm} = 0.0100 \text{ kg/m}$

$$f_n = \frac{nv}{2L}$$

where

$$v = \left(\frac{T}{\mu}\right)^{1/2} = 70.7 \text{ m/s}$$

$$f_n = n \left(\frac{70.7}{1.20}\right) = 58.9n = 20\,000 \text{ Hz}$$

Largest $n = 339 \Rightarrow f = \boxed{19.976 \text{ kHz}}$.

P18.27 $d_{NN} = 0.700 \text{ m}$

$\lambda = 1.40 \text{ m}$

$$f\lambda = v = 308 \text{ m/s} = \sqrt{\frac{T}{(1.20 \times 10^{-3})/(0.700)}}$$

(a) $T = \boxed{163 \text{ N}}$

(b) $f_3 = \boxed{660 \text{ Hz}}$

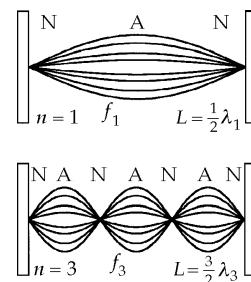


FIG. P18.27

P18.28 $\lambda_G = 2(0.350 \text{ m}) = \frac{v}{f_G}; \lambda_A = 2L_A = \frac{v}{f_A}$

$$L_G - L_A = L_G - \left(\frac{f_G}{f_A}\right)L_G = L_G \left(1 - \frac{f_G}{f_A}\right) = (0.350 \text{ m}) \left(1 - \frac{392}{440}\right) = 0.0382 \text{ m}$$

Thus, $L_A = L_G - 0.0382 \text{ m} = 0.350 \text{ m} - 0.0382 \text{ m} = 0.312 \text{ m}$,

or the finger should be placed 31.2 cm from the bridge.

$$L_A = \frac{v}{2f_A} = \frac{1}{2f_A} \sqrt{\frac{T}{\mu}}; dL_A = \frac{dT}{4f_A \sqrt{T\mu}}; \frac{dL_A}{L_A} = \frac{1}{2} \frac{dT}{T}$$

$$\frac{dT}{T} = 2 \frac{dL_A}{L_A} = 2 \frac{0.600 \text{ cm}}{(35.0 - 3.82) \text{ cm}} = \boxed{3.84\%}$$

P18.29 In the fundamental mode, the string above the rod has only two nodes, at A and B, with an anti-node halfway between A and B. Thus,

$$\frac{\lambda}{2} = \overline{AB} = \frac{L}{\cos \theta} \text{ or } \lambda = \frac{2L}{\cos \theta}.$$

Since the fundamental frequency is f , the wave speed in this segment of string is

$$v = \lambda f = \frac{2Lf}{\cos \theta}.$$

Also, $v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{T}{m/\overline{AB}}} = \sqrt{\frac{TL}{m \cos \theta}}$

where T is the tension in this part of the string. Thus,

$$\frac{2Lf}{\cos \theta} = \sqrt{\frac{TL}{m \cos \theta}} \text{ or } \frac{4L^2 f^2}{\cos^2 \theta} = \frac{TL}{m \cos \theta}$$

and the mass of string above the rod is:

$$m = \frac{T \cos \theta}{4Lf^2} \quad [\text{Equation 1}]$$

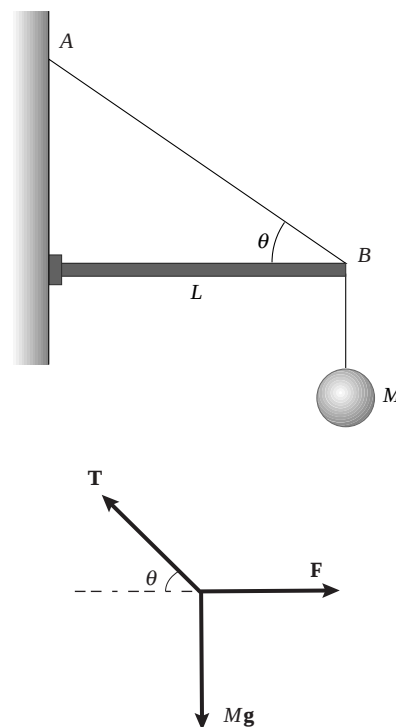


FIG. P18.29

Now, consider the tension in the string. The light rod would rotate about point P if the string exerted any vertical force on it. Therefore, recalling Newton's third law, the rod must exert only a horizontal force on the string. Consider a free-body diagram of the string segment in contact with the end of the rod.

$$\sum F_y = T \sin \theta - Mg = 0 \Rightarrow T = \frac{Mg}{\sin \theta}$$

Then, from Equation 1, the mass of string above the rod is

$$m = \left(\frac{Mg}{\sin \theta}\right) \frac{\cos \theta}{4Lf^2} = \boxed{\frac{Mg}{4Lf^2 \tan \theta}}.$$

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***P18.30** Let $m = \rho V$ represent the mass of the copper cylinder. The original tension in the wire is $T_1 = mg = \rho Vg$. The water exerts a buoyant force $\rho_{\text{water}}\left(\frac{V}{2}\right)g$ on the cylinder, to reduce the tension to

$$T_2 = \rho Vg - \rho_{\text{water}}\left(\frac{V}{2}\right)g = \left(\rho - \frac{\rho_{\text{water}}}{2}\right)Vg.$$

The speed of a wave on the string changes from $\sqrt{\frac{T_1}{\mu}}$ to $\sqrt{\frac{T_2}{\mu}}$. The frequency changes from

$$f_1 = \frac{v_1}{\lambda} = \sqrt{\frac{T_1}{\mu}} \frac{1}{\lambda} \text{ to } f_2 = \sqrt{\frac{T_2}{\mu}} \frac{1}{\lambda}$$

where we assume $\lambda = 2L$ is constant.

Then
$$\frac{f_2}{f_1} = \sqrt{\frac{T_2}{T_1}} = \sqrt{\frac{\rho - \rho_{\text{water}}/2}{\rho}} = \sqrt{\frac{8.92 - 1.00/2}{8.92}}$$

$$f_2 = 300 \text{ Hz} \sqrt{\frac{8.42}{8.92}} = \boxed{291 \text{ Hz}}$$

***P18.31** Comparing $y = (0.002 \text{ m})\sin((\pi \text{ rad/m})x)\cos((100\pi \text{ rad/s})t)$
with $y = 2A \sin kx \cos \omega t$
we find $k = \frac{2\pi}{\lambda} = \pi \text{ m}^{-1}$, $\lambda = 2.00 \text{ m}$, and $\omega = 2\pi f = 100\pi \text{ s}^{-1}$: $f = 50.0 \text{ Hz}$

(a) Then the distance between adjacent nodes is $d_{\text{NN}} = \frac{\lambda}{2} = 1.00 \text{ m}$
and on the string are $\frac{L}{d_{\text{NN}}} = \frac{3.00 \text{ m}}{1.00 \text{ m}} = \boxed{3 \text{ loops}}$
For the speed we have $v = f\lambda = (50 \text{ s}^{-1})2 \text{ m} = 100 \text{ m/s}$

(b) In the simplest standing wave vibration, $d_{\text{NN}} = 3.00 \text{ m} = \frac{\lambda_b}{2}$, $\lambda_b = 6.00 \text{ m}$
and $f_b = \frac{v_a}{\lambda_b} = \frac{100 \text{ m/s}}{6.00 \text{ m}} = \boxed{16.7 \text{ Hz}}$

(c) In $v_0 = \sqrt{\frac{T_0}{\mu}}$, if the tension increases to $T_c = 9T_0$ and the string does not stretch, the speed increases to

$$v_c = \sqrt{\frac{9T_0}{\mu}} = 3\sqrt{\frac{T_0}{\mu}} = 3v_0 = 3(100 \text{ m/s}) = 300 \text{ m/s}$$

Then $\lambda_c = \frac{v_c}{f_a} = \frac{300 \text{ m/s}}{50 \text{ s}^{-1}} = 6.00 \text{ m}$ $d_{\text{NN}} = \frac{\lambda_c}{2} = 3.00 \text{ m}$

and $\boxed{\text{one}}$ loop fits onto the string.

Section 18.4 Resonance

P18.32 The natural frequency is

$$f = \frac{1}{T} = \frac{1}{2\pi} \sqrt{\frac{g}{L}} = \frac{1}{2\pi} \sqrt{\frac{9.80 \text{ m/s}^2}{2.00 \text{ m}}} = 0.352 \text{ Hz}.$$

The big brother must push at this same frequency of 0.352 Hz to produce resonance.

P18.33 (a) The wave speed is $v = \frac{9.15 \text{ m}}{2.50 \text{ s}} = \text{3.66 m/s}$

(b) From the figure, there are antinodes at both ends of the pond, so the distance between adjacent antinodes

is $d_{AA} = \frac{\lambda}{2} = 9.15 \text{ m},$

and the wavelength is $\lambda = 18.3 \text{ m}$

The frequency is then $f = \frac{v}{\lambda} = \frac{3.66 \text{ m/s}}{18.3 \text{ m}} = \text{0.200 Hz}$

We have assumed the wave speed is the same for all wavelengths.

P18.34 The wave speed is $v = \sqrt{gd} = \sqrt{(9.80 \text{ m/s}^2)(36.1 \text{ m})} = 18.8 \text{ m/s}$

The bay has one end open and one closed. Its simplest resonance is with a node of horizontal velocity, which is also an antinode of vertical displacement, at the head of the bay and an antinode of velocity, which is a node of displacement, at the mouth. The vibration of the water in the bay is like that in one half of the pond shown in Figure P18.33.

Then, $d_{NA} = 210 \times 10^3 \text{ m} = \frac{\lambda}{4}$

and $\lambda = 840 \times 10^3 \text{ m}$

Therefore, the period is $T = \frac{1}{f} = \frac{\lambda}{v} = \frac{840 \times 10^3 \text{ m}}{18.8 \text{ m/s}} = 4.47 \times 10^4 \text{ s} = \text{12 h 24 min}$

This agrees precisely with the period of the lunar excitation, so we identify the extra-high tides as amplified by resonance.

P18.35 The distance between adjacent nodes is one-quarter of the circumference.

$$d_{NN} = d_{AA} = \frac{\lambda}{2} = \frac{20.0 \text{ cm}}{4} = 5.00 \text{ cm}$$

so $\lambda = 10.0 \text{ cm}$ and $f = \frac{v}{\lambda} = \frac{900 \text{ m/s}}{0.100 \text{ m}} = 9\,000 \text{ Hz} = \text{9.00 kHz}$.

The singer must match this frequency quite precisely for some interval of time to feed enough energy into the glass to crack it.

Section 18.5 Standing Waves in Air Columns

P18.36 $d_{AA} = 0.320 \text{ m}$; $\lambda = 0.640 \text{ m}$

(a) $f = \frac{v}{\lambda} = \boxed{531 \text{ Hz}}$

(b) $\lambda = 0.0850 \text{ m}$; $d_{AA} = \boxed{42.5 \text{ mm}}$

P18.37 (a) For the fundamental mode in a closed pipe, $\lambda = 4L$, as in the diagram.

But $v = f\lambda$, therefore $L = \frac{v}{4f}$

So, $L = \frac{343 \text{ m/s}}{4(240 \text{ s}^{-1})} = \boxed{0.357 \text{ m}}$

(b) For an open pipe, $\lambda = 2L$, as in the diagram.

So, $L = \frac{v}{2f} = \frac{343 \text{ m/s}}{2(240 \text{ s}^{-1})} = \boxed{0.715 \text{ m}}$

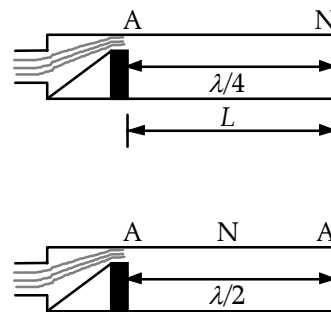


FIG. P18.37

P18.38 The wavelength is $\lambda = \frac{v}{f} = \frac{343 \text{ m/s}}{261.6/\text{s}} = 1.31 \text{ m}$

so the length of the open pipe vibrating in its simplest (A-N-A) mode is

$$d_{A \text{ to } A} = \frac{1}{2} \lambda = \boxed{0.656 \text{ m}}$$

A closed pipe has (N-A) for its simplest resonance,

(N-A-N-A) for the second,

and (N-A-N-A-N-A) for the third.

Here, the pipe length is $5d_{N \text{ to } A} = \frac{5\lambda}{4} = \frac{5}{4}(1.31 \text{ m}) = \boxed{1.64 \text{ m}}$

***P18.39** Assuming an air temperature of $T = 37^\circ\text{C} = 310 \text{ K}$, the speed of sound inside the pipe is

$$v = (331 \text{ m/s}) \sqrt{\frac{310 \text{ K}}{273 \text{ K}}} = 353 \text{ m/s}.$$

In the fundamental resonant mode, the wavelength of sound waves in a pipe closed at one end is $\lambda = 4L$. Thus, for the whooping crane

$$\lambda = 4(5.0 \text{ ft}) = 2.0 \times 10^1 \text{ ft} \text{ and } f = \frac{v}{\lambda} = \frac{(353 \text{ m/s})}{2.0 \times 10^1 \text{ ft} \left(\frac{3.281 \text{ ft}}{1 \text{ m}} \right)} = \boxed{57.9 \text{ Hz}}.$$

- P18.40** The air in the auditory canal, about 3 cm long, can vibrate with a node at the closed end and antinode at the open end,

$$\text{with } d_{N \text{ to } A} = 3 \text{ cm} = \frac{\lambda}{4}$$

$$\text{so } \lambda = 0.12 \text{ m}$$

$$\text{and } f = \frac{v}{\lambda} = \frac{343 \text{ m/s}}{0.12 \text{ m}} \approx \boxed{3 \text{ kHz}}$$

A small-amplitude external excitation at this frequency can, over time, feed energy into a larger-amplitude resonance vibration of the air in the canal, making it audible.

- P18.41** For a closed box, the resonant frequencies will have nodes at both sides, so the permitted wavelengths will be $L = \frac{1}{2}n\lambda$, ($n = 1, 2, 3, \dots$).

$$\text{i.e., } L = \frac{n\lambda}{2} = \frac{nv}{2f} \text{ and } f = \frac{nv}{2L}.$$

Therefore, with $L = 0.860 \text{ m}$ and $L' = 2.10 \text{ m}$, the resonant frequencies are

$$f_n = \boxed{n(206 \text{ Hz})} \text{ for } L = 0.860 \text{ m for each } n \text{ from 1 to 9}$$

$$\text{and } f'_n = \boxed{n(84.5 \text{ Hz})} \text{ for } L' = 2.10 \text{ m for each } n \text{ from 2 to 23.}$$

- P18.42** The wavelength of sound is

$$\lambda = \frac{v}{f}$$

$$\text{The distance between water levels at resonance is } d = \frac{v}{2f} \quad \therefore Rt = \pi r^2 d = \frac{\pi r^2 v}{2f}$$

and

$$t = \boxed{\frac{\pi r^2 v}{2Rf}}.$$

- P18.43** For both open and closed pipes, resonant frequencies are equally spaced as numbers. The set of resonant frequencies then must be 650 Hz, 550 Hz, 450 Hz, 350 Hz, 250 Hz, 150 Hz, 50 Hz. These are odd-integer multipliers of the fundamental frequency of $\boxed{50.0 \text{ Hz}}$. Then the pipe length is

$$d_{NA} = \frac{\lambda}{4} = \frac{v}{4f} = \frac{340 \text{ m/s}}{4(50/\text{s})} = \boxed{1.70 \text{ m}}.$$

$$\text{P18.44} \quad \frac{\lambda}{2} = d_{AA} = \frac{L}{n} \text{ or } L = \frac{n\lambda}{2} \quad \text{for } n = 1, 2, 3, \dots$$

$$\text{Since } \lambda = \frac{v}{f} \quad L = n \left(\frac{v}{2f} \right) \quad \text{for } n = 1, 2, 3, \dots$$

With $v = 343 \text{ m/s}$ and

$$f = 680 \text{ Hz},$$

$$L = n \left(\frac{343 \text{ m/s}}{2(680 \text{ Hz})} \right) = n(0.252 \text{ m}) \quad \text{for } n = 1, 2, 3, \dots$$

Possible lengths for resonance are: $L = \boxed{0.252 \text{ m}, 0.504 \text{ m}, 0.757 \text{ m}, \dots, n(0.252) \text{ m}}.$

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P18.45 For resonance in a narrow tube open at one end,

$$f = n \frac{v}{4L} \quad (n = 1, 3, 5, \dots).$$

(a) Assuming $n = 1$ and $n = 3$,

$$384 = \frac{v}{4(0.228)} \quad \text{and} \quad 384 = \frac{3v}{4(0.683)}.$$

In either case, $v = \boxed{350 \text{ m/s}}$.

(b) For the next resonance $n = 5$, and $L = \frac{5v}{4f} = \frac{5(350 \text{ m/s})}{4(384 \text{ s}^{-1})} = \boxed{1.14 \text{ m}}$.

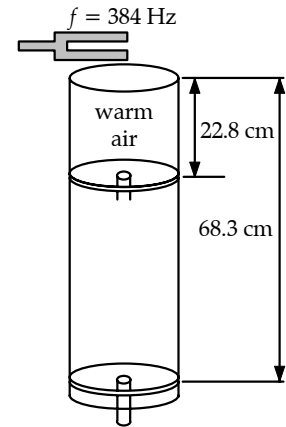


FIG. P18.45

P18.46 The length corresponding to the fundamental satisfies $f = \frac{v}{4L}$: $L_1 = \frac{v}{4f} = \frac{34}{4(512)} = 0.167 \text{ m}$.

Since $L > 20.0 \text{ cm}$, the *next* two modes will be observed, corresponding to $f = \frac{3v}{4L_2}$ and $f = \frac{5v}{4L_3}$.

or $L_2 = \frac{3v}{4f} = \boxed{0.502 \text{ m}}$ and $L_3 = \frac{5v}{4f} = \boxed{0.837 \text{ m}}$.

P18.47 We suppose these are the lowest resonances of the enclosed air columns.

For one, $\lambda = \frac{v}{f} = \frac{343 \text{ m/s}}{256 \text{ s}^{-1}} = 1.34 \text{ m}$ length $= d_{AA} = \frac{\lambda}{2} = 0.670 \text{ m}$

For the other, $\lambda = \frac{v}{f} = \frac{343 \text{ m/s}}{440 \text{ s}^{-1}} = 0.780 \text{ m}$ length $= 0.390 \text{ m}$

So,

(b) original length $= \boxed{1.06 \text{ m}}$

$$\lambda = 2d_{AA} = 2.12 \text{ m}$$

(a) $f = \frac{343 \text{ m/s}}{2.12 \text{ m}} = \boxed{162 \text{ Hz}}$

P18.48 (a) For the fundamental mode of an open tube,

$$L = \frac{\lambda}{2} = \frac{v}{2f} = \frac{343 \text{ m/s}}{2(880 \text{ s}^{-1})} = \boxed{0.195 \text{ m}}.$$

(b) $v = 331 \text{ m/s} \sqrt{1 + \frac{(-5.00)}{273}} = 328 \text{ m/s}$

We ignore the thermal expansion of the metal.

$$f = \frac{v}{\lambda} = \frac{v}{2L} = \frac{328 \text{ m/s}}{2(0.195 \text{ m})} = \boxed{841 \text{ Hz}}$$

The flute is flat by a semitone.

Section 18.6 Standing Waves in Rod and Plates

P18.49 (a) $f = \frac{v}{2L} = \frac{5\,100}{(2)(1.60)} = \boxed{1.59 \text{ kHz}}$

- (b) Since it is held in the center, there must be a node in the center as well as antinodes at the ends. The even harmonics have an antinode at the center so only the odd harmonics are present.

(c) $f = \frac{v'}{2L} = \frac{3\,560}{(2)(1.60)} = \boxed{1.11 \text{ kHz}}$

P18.50 When the rod is clamped at one-quarter of its length, the vibration pattern reads ANANA and the rod length is $L = 2d_{AA} = \lambda$.

Therefore, $L = \frac{v}{f} = \frac{5\,100 \text{ m/s}}{4\,400 \text{ Hz}} = \boxed{1.16 \text{ m}}$

Section 18.7 Beats: Interference in Time

P18.51 $f \propto v \propto \sqrt{T}$ $f_{\text{new}} = 110 \sqrt{\frac{540}{600}} = 104.4 \text{ Hz}$

$\Delta f = \boxed{5.64 \text{ beats/s}}$

P18.52 (a) The string could be tuned to either 521 Hz or 525 Hz from this evidence.

- (b) Tightening the string raises the wave speed and frequency. If the frequency were originally 521 Hz, the beats would slow down.

Instead, the frequency must have started at 525 Hz to become 526 Hz.

(c) From $f = \frac{v}{\lambda} = \frac{\sqrt{T/\mu}}{2L} = \frac{1}{2L} \sqrt{\frac{T}{\mu}}$

$$\frac{f_2}{f_1} = \sqrt{\frac{T_2}{T_1}} \text{ and } T_2 = \left(\frac{f_2}{f_1}\right)^2 T_1 = \left(\frac{523 \text{ Hz}}{526 \text{ Hz}}\right)^2 T_1 = 0.989 T_1.$$

The fractional change that should be made in the tension is then

$$\text{fractional change} = \frac{T_1 - T_2}{T_1} = 1 - 0.989 = 0.0114 = 1.14\% \text{ lower.}$$

The tension should be reduced by 1.14%.

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P18.53 For an echo $f' = f \frac{(v + v_s)}{(v - v_s)}$ the beat frequency is $f_b = |f' - f|$.

Solving for f_b .

gives $f_b = f \frac{(2v_s)}{(v - v_s)}$ when approaching wall.

$$(a) \quad f_b = (256) \frac{2(1.33)}{(343 - 1.33)} = \boxed{1.99 \text{ Hz}} \text{ beat frequency}$$

(b) When he is moving away from the wall, v_s changes sign. Solving for v_s gives

$$v_s = \frac{f_b v}{2f - f_b} = \frac{(5)(343)}{(2)(256) - 5} = \boxed{3.38 \text{ m/s}}.$$

***P18.54** Using the $\boxed{4 \text{ and } 2\frac{2}{3} \text{ - foot pipes}}$ produces actual frequencies of 131 Hz and 196 Hz and a combination tone at $(196 - 131)\text{Hz} = 65.4 \text{ Hz}$, so this pair supplies the so-called missing fundamental. The 4 and 2-foot pipes produce a combination tone $(262 - 131)\text{Hz} = 131 \text{ Hz}$, so this does not work.

The $\boxed{2\frac{2}{3} \text{ and } 2 \text{ - foot pipes}}$ produce a combination tone at $(262 - 196)\text{Hz} = 65.4 \text{ Hz}$, so this works.

Also, $\boxed{4, 2\frac{2}{3}, \text{ and } 2 \text{ - foot pipes}}$ all playing together produce the 65.4-Hz combination tone.

Section 18.8 Non-Sinusoidal Wave Patterns

P18.55 We list the frequencies of the harmonics of each note in Hz:

Note	Harmonic				
	1	2	3	4	5
A	440.00	880.00	1 320.0	1 760.0	2 200.0
C#	554.37	1 108.7	1 663.1	2 217.5	2 771.9
E	659.26	1 318.5	1 977.8	2 637.0	3 296.3

The second harmonic of E is close the the third harmonic of A, and the fourth harmonic of C# is close to the fifth harmonic of A.

P18.56 We evaluate

$$s = 100 \sin \theta + 157 \sin 2\theta + 62.9 \sin 3\theta + 105 \sin 4\theta \\ + 51.9 \sin 5\theta + 29.5 \sin 6\theta + 25.3 \sin 7\theta$$

where s represents particle displacement in nanometers and θ represents the phase of the wave in radians. As θ advances by 2π , time advances by $(1/523) \text{ s}$. Here is the result:

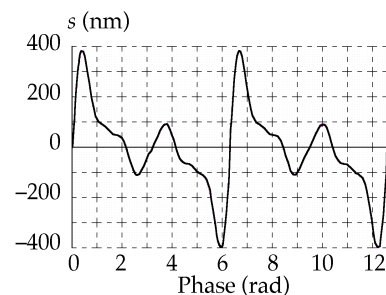


FIG. P18.56

Additional Problems

P18.57 $f = 87.0 \text{ Hz}$

speed of sound in air: $v_a = 340 \text{ m/s}$

(a) $\lambda_b = \ell \quad v = f\lambda_b = (87.0 \text{ s}^{-1})(0.400 \text{ m})$

$$v = \boxed{34.8 \text{ m/s}}$$

(b) $\left. \begin{array}{l} \lambda_a = 4L \\ v_a = \lambda_a f \end{array} \right\} \quad L = \frac{v_a}{4f} = \frac{340 \text{ m/s}}{4(87.0 \text{ s}^{-1})} = \boxed{0.977 \text{ m}}$

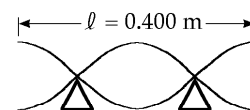


FIG. P18.57

***P18.58** (a) Use the Doppler formula

$$f' = f \frac{(v \pm v_0)}{(v \mp v_s)}$$

With f'_1 = frequency of the speaker in front of student and

f'_2 = frequency of the speaker behind the student.

$$f'_1 = (456 \text{ Hz}) \frac{(343 \text{ m/s} + 1.50 \text{ m/s})}{(343 \text{ m/s} - 0)} = 458 \text{ Hz}$$

$$f'_2 = (456 \text{ Hz}) \frac{(343 \text{ m/s} - 1.50 \text{ m/s})}{(343 \text{ m/s} + 0)} = 454 \text{ Hz}$$

Therefore, $f_b = f'_1 - f'_2 = \boxed{3.99 \text{ Hz}}$.

(b) The waves broadcast by both speakers have $\lambda = \frac{v}{f} = \frac{343 \text{ m/s}}{456/\text{s}} = 0.752 \text{ m}$. The standing wave

between them has $d_{AA} = \frac{\lambda}{2} = 0.376 \text{ m}$. The student walks from one maximum to the next in

time $\Delta t = \frac{0.376 \text{ m}}{1.50 \text{ m/s}} = 0.251 \text{ s}$, so the frequency at which she hears maxima is $f = \frac{1}{T} = \boxed{3.99 \text{ Hz}}$.

P18.59 Moving away from station, frequency is depressed:

$$f' = 180 - 2.00 = 178 \text{ Hz}: \quad 178 = 180 \frac{343}{343 - (-v)}$$

Solving for v gives
$$v = \frac{(2.00)(343)}{178}$$

Therefore,
$$v = \boxed{3.85 \text{ m/s away from station}}$$

Moving toward the station, the frequency is enhanced:

$$f' = 180 + 2.00 = 182 \text{ Hz}: \quad 182 = 180 \frac{343}{343 - v}$$

Solving for v gives
$$4 = \frac{(2.00)(343)}{182}$$

Therefore,
$$v = \boxed{3.77 \text{ m/s toward the station}}$$

544 Superposition and Standing Waves

P18.60 $v = \sqrt{\frac{(48.0)(2.00)}{4.80 \times 10^{-3}}} = 141 \text{ m/s}$

$d_{\text{NN}} = 1.00 \text{ m}; \lambda = 2.00 \text{ m}; f = \frac{v}{\lambda} = 70.7 \text{ Hz}$

$\lambda_a = \frac{v_a}{f} = \frac{343 \text{ m/s}}{70.7 \text{ Hz}} = \boxed{4.85 \text{ m}}$

P18.61 Call L the depth of the well and v the speed of sound.

Then for some integer n
$$L = (2n-1) \frac{\lambda_1}{4} = (2n-1) \frac{v}{4f_1} = \frac{(2n-1)(343 \text{ m/s})}{4(51.5 \text{ s}^{-1})}$$

and for the next resonance
$$L = [2(n+1)-1] \frac{\lambda_2}{4} = (2n+1) \frac{v}{4f_2} = \frac{(2n+1)(343 \text{ m/s})}{4(60.0 \text{ s}^{-1})}$$

Thus,
$$\frac{(2n-1)(343 \text{ m/s})}{4(51.5 \text{ s}^{-1})} = \frac{(2n+1)(343 \text{ m/s})}{4(60.0 \text{ s}^{-1})}$$

and we require an *integer* solution to
$$\frac{2n+1}{60.0} = \frac{2n-1}{51.5}$$

The equation gives $n = \frac{111.5}{17} = 6.56$, so the best fitting integer is $n = 7$.

Then
$$L = \frac{[2(7)-1](343 \text{ m/s})}{4(51.5 \text{ s}^{-1})} = 21.6 \text{ m}$$

and
$$L = \frac{[2(7)+1](343 \text{ m/s})}{4(60.0 \text{ s}^{-1})} = 21.4 \text{ m}$$

suggest the best value for the depth of the well is $\boxed{21.5 \text{ m}}$.

P18.62 The second standing wave mode of the air in the pipe reads ANAN, with $d_{\text{NA}} = \frac{\lambda}{4} = \frac{1.75 \text{ m}}{3}$

so $\lambda = 2.33 \text{ m}$

and $f = \frac{v}{\lambda} = \frac{343 \text{ m/s}}{2.33 \text{ m}} = 147 \text{ Hz}$

For the string, λ and v are different but f is the same.

$$\frac{\lambda}{2} = d_{\text{NN}} = \frac{0.400 \text{ m}}{2}$$

so $\lambda = 0.400 \text{ m}$

$$v = \lambda f = (0.400 \text{ m})(147 \text{ Hz}) = 58.8 \text{ m/s} = \sqrt{\frac{T}{\mu}}$$

$$T = \mu v^2 = (9.00 \times 10^{-3} \text{ kg/m})(58.8 \text{ m/s})^2 = \boxed{31.1 \text{ N}}$$

- P18.63** (a) Since the first node is at the weld, the wavelength in the thin wire is $2L$ or 80.0 cm. The frequency and tension are the same in both sections, so

$$f = \frac{1}{2L} \sqrt{\frac{T}{\mu}} = \frac{1}{2(0.400)} \sqrt{\frac{4.60}{2.00 \times 10^{-3}}} = \boxed{59.9 \text{ Hz}}.$$

- (b) As the thick wire is twice the diameter, the linear density is 4 times that of the thin wire.

$$\mu' = 8.00 \text{ g/m}$$

$$\text{so } L' = \frac{1}{2f} \sqrt{\frac{T}{\mu'}} \quad L' = \left[\frac{1}{(2)(59.9)} \right] \sqrt{\frac{4.60}{8.00 \times 10^{-3}}} = \boxed{20.0 \text{ cm}} \text{ half the length of the thin wire.}$$

- P18.64** (a) For the block:

$$\sum F_x = T - Mg \sin 30.0^\circ = 0$$

$$\text{so } T = Mg \sin 30.0^\circ = \boxed{\frac{1}{2} Mg}.$$

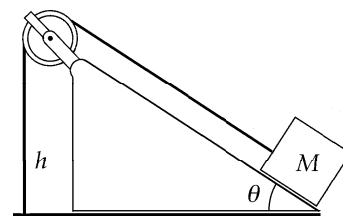


FIG. P18.64

- (b) The length of the section of string parallel to the incline is $\frac{h}{\sin 30.0^\circ} = 2h$. The total length of the string is then $\boxed{3h}$.

- (c) The mass per unit length of the string is $\mu = \boxed{\frac{m}{3h}}$

- (d) The speed of waves in the string is $v = \sqrt{\frac{T}{\mu}} = \sqrt{\left(\frac{Mg}{2}\right)\left(\frac{3h}{m}\right)} = \boxed{\sqrt{\frac{3Mgh}{2m}}}$

- (e) In the fundamental mode, the segment of length h vibrates as one loop. The distance between adjacent nodes is then $d_{NN} = \frac{\lambda}{2} = h$, so the wavelength is $\lambda = 2h$.

$$\text{The frequency is} \quad f = \frac{v}{\lambda} = \frac{1}{2h} \sqrt{\frac{3Mgh}{2m}} = \boxed{\sqrt{\frac{3Mg}{8mh}}}$$

- (g) When the vertical segment of string vibrates with 2 loops (i.e., 3 nodes), then $h = 2\left(\frac{\lambda}{2}\right)$ and the wavelength is $\lambda = \boxed{h}$.

- (f) The period of the standing wave of 3 nodes (or two loops) is

$$T = \frac{1}{f} = \frac{\lambda}{v} = h \sqrt{\frac{2m}{3Mgh}} = \boxed{\sqrt{\frac{2mh}{3Mg}}}$$

- (h) $f_b = 1.02f - f = (2.00 \times 10^{-2})f = \boxed{(2.00 \times 10^{-2}) \sqrt{\frac{3Mg}{8mh}}}$

P18.65 (a) $f = \frac{n}{2L} \sqrt{\frac{T}{\mu}}$
 so $\frac{f'}{f} = \frac{L}{L'} = \frac{L}{2L} = \frac{1}{2}$

The frequency should be halved to get the same number of antinodes for twice the length.

(b) $\frac{n'}{n} = \sqrt{\frac{T}{T'}}$ so $\frac{T'}{T} = \left(\frac{n}{n'}\right)^2 = \left[\frac{n}{n+1}\right]^2$

The tension must be $T' = \left[\frac{n}{n+1}\right]^2 T$

(c) $\frac{f'}{f} = \frac{n'L}{nL'} \sqrt{\frac{T'}{T}}$ so $\frac{T'}{T} = \left(\frac{nfL'}{n'fL}\right)^2$
 $\frac{T'}{T} = \left(\frac{3}{2 \cdot 2}\right)^2$ $\frac{T'}{T} = \frac{9}{16}$ to get twice as many antinodes.

P18.66 For the wire, $\mu = \frac{0.0100 \text{ kg}}{2.00 \text{ m}} = 5.00 \times 10^{-3} \text{ kg/m}$: $v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{(200 \text{ kg} \cdot \text{m/s}^2)}{5.00 \times 10^{-3} \text{ kg/m}}}$
 $v = 200 \text{ m/s}$

If it vibrates in its simplest state, $d_{\text{NN}} = 2.00 \text{ m} = \frac{\lambda}{2}$: $f = \frac{v}{\lambda} = \frac{(200 \text{ m/s})}{4.00 \text{ m}} = 50.0 \text{ Hz}$

(a) The tuning fork can have frequencies 45.0 Hz or 55.0 Hz.

(b) If $f = 45.0 \text{ Hz}$, $v = f\lambda = (45.0/\text{s})(4.00 \text{ m}) = 180 \text{ m/s}$.

Then, $T = v^2 \mu = (180 \text{ m/s})^2 (5.00 \times 10^{-3} \text{ kg/m}) = \text{162 N}$

or if $f = 55.0 \text{ Hz}$, $T = v^2 \mu = f^2 \lambda^2 \mu = (55.0/\text{s})^2 (4.00 \text{ m})^2 (5.00 \times 10^{-3} \text{ kg/m}) = \text{242 N}.$

P18.67 We look for a solution of the form

$$5.00 \sin(2.00x - 10.0t) + 10.0 \cos(2.00x - 10.0t) = A \sin(2.00x - 10.0t + \phi)$$

$$= A \sin(2.00x - 10.0t) \cos \phi + A \cos(2.00x - 10.0t) \sin \phi$$

This will be true if both $5.00 = A \cos \phi$ and $10.0 = A \sin \phi$,

requiring $(5.00)^2 + (10.0)^2 = A^2$

$A = 11.2$ and $\phi = 63.4^\circ$

The resultant wave $11.2 \sin(2.00x - 10.0t + 63.4^\circ)$ is sinusoidal.

P18.68 (a) With $k = \frac{2\pi}{\lambda}$ and $\omega = 2\pi f = \frac{2\pi v}{\lambda}$: $y(x, t) = 2A \sin kx \cos \omega t = \boxed{2A \sin\left(\frac{2\pi x}{\lambda}\right) \cos\left(\frac{2\pi vt}{\lambda}\right)}$

(b) For the fundamental vibration, $\lambda_1 = 2L$

so $y_1(x, t) = \boxed{2A \sin\left(\frac{\pi x}{L}\right) \cos\left(\frac{\pi vt}{L}\right)}$

(c) For the second harmonic $\lambda_2 = L$ and $y_2(x, t) = \boxed{2A \sin\left(\frac{2\pi x}{L}\right) \cos\left(\frac{2\pi vt}{L}\right)}$

(d) In general, $\lambda_n = \frac{2L}{n}$ and $y_n(x, t) = \boxed{2A \sin\left(\frac{n\pi x}{L}\right) \cos\left(\frac{n\pi vt}{L}\right)}$

P18.69 (a) Let θ represent the angle each slanted rope makes with the vertical.

In the diagram, observe that:

$$\sin \theta = \frac{1.00 \text{ m}}{1.50 \text{ m}} = \frac{2}{3}$$

or $\theta = 41.8^\circ$.

Considering the mass,

$$\sum F_y = 0: 2T \cos \theta = mg$$

$$\text{or } T = \frac{(12.0 \text{ kg})(9.80 \text{ m/s}^2)}{2 \cos 41.8^\circ} = \boxed{78.9 \text{ N}}$$

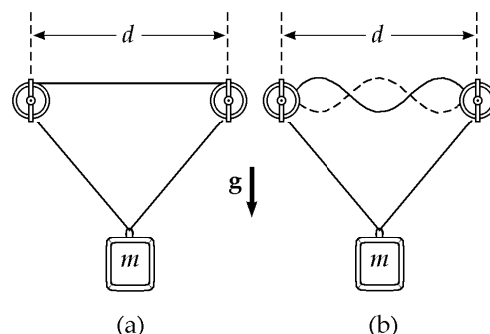


FIG. P18.69

(b) The speed of transverse waves in the string is $v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{78.9 \text{ N}}{0.00100 \text{ kg/m}}} = 281 \text{ m/s}$

For the standing wave pattern shown (3 loops), $d = \frac{3}{2} \lambda$

or $\lambda = \frac{2(2.00 \text{ m})}{3} = 1.33 \text{ m}$

Thus, the required frequency is $f = \frac{v}{\lambda} = \frac{281 \text{ m/s}}{1.33 \text{ m}} = \boxed{211 \text{ Hz}}$

***P18.70** $d_{AA} = \frac{\lambda}{2} = 7.05 \times 10^{-3} \text{ m}$ is the distance between antinodes.

Then $\lambda = 14.1 \times 10^{-3} \text{ m}$

$$\text{and } f = \frac{v}{\lambda} = \frac{3.70 \times 10^3 \text{ m/s}}{14.1 \times 10^{-3} \text{ m}} = \boxed{2.62 \times 10^5 \text{ Hz}}.$$

The crystal can be tuned to vibrate at 2^{18} Hz , so that binary counters can derive from it a signal at precisely 1 Hz .

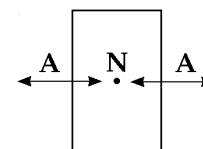


FIG. P18.70

ANSWERS TO EVEN PROBLEMS

- P18.2** see the solution
- P18.4** 5.66 cm
- P18.6** 0.500 s
- P18.8** (a) 3.33 rad; (b) 283 Hz
- P18.10** (a) The number is the greatest integer $\leq d\left(\frac{f}{v}\right) + \frac{1}{2}$;
 (b) $L_n = \frac{d^2 - (n - 1/2)^2 (v/f)^2}{2(n - 1/2)(v/f)}$ where $n = 1, 2, \dots, n_{\max}$
- P18.12** (a) $\Delta x = \frac{\lambda}{2}$;
 (b) along the hyperbola $9x^2 - 16y^2 = 144$
- P18.14** (a) $(2n + 1)\pi$ m for $n = 0, 1, 2, 3, \dots$;
 (b) 0.029 4 m
- P18.16** see the solution
- P18.18** see the solution
- P18.20** 15.7 Hz
- P18.22** (a) 257 Hz; (b) 6
- P18.24** (a) 495 Hz; (b) 990 Hz
- P18.26** 19.976 kHz
- P18.28** 3.84%
- P18.30** 291 Hz
- P18.32** 0.352 Hz
- P18.34** see the solution
- P18.36** (a) 531 Hz; (b) 42.5 mm
- P18.38** 0.656 m; 1.64 m
- P18.40** 3 kHz; see the solution
- P18.42** $\Delta t = \frac{\pi r^2 v}{2Rf}$
- P18.44** $L = 0.252$ m, 0.504 m, 0.757 m, ..., $n(0.252)$ m for $n = 1, 2, 3, \dots$
- P18.46** 0.502 m; 0.837 m
- P18.48** (a) 0.195 m; (b) 841 m
- P18.50** 1.16 m
- P18.52** (a) 521 Hz or 525 Hz; (b) 526 Hz;
 (c) reduce by 1.14%
- P18.54** 4-foot and $2\frac{2}{3}$ -foot; $2\frac{2}{3}$ and 2-foot; and all three together
- P18.56** see the solution
- P18.58** (a) and (b) 3.99 beats/s
- P18.60** 4.85 m
- P18.62** 31.1 N
- P18.64** (a) $\frac{1}{2}Mg$; (b) $3h$; (c) $\frac{m}{3h}$; (d) $\sqrt{\frac{3Mgh}{2m}}$;
 (e) $\sqrt{\frac{3Mg}{8mh}}$; (f) $\sqrt{\frac{2mh}{3Mg}}$; (g) h ;
 (h) $(2.00 \times 10^{-2})\sqrt{\frac{3Mg}{8mh}}$
- P18.66** (a) 45.0 Hz or 55.0 Hz; (b) 162 N or 242 N
- P18.68** see the solution
- P18.70** 262 kHz